RELATIVISTIC QUANTUM LOGIC
AND
THE DYNAMICS OF NATURAL LANGUAGES*

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* or "THE WARP HYPOTHESIS II"

Errata

	forward	line 7	.....also desirable that...
		line 16	...McNeill, as we.....
		line 19	...this paper will not,in all....
	page 1	line 3	In general, one....
par.2 {	page 2	line 6	....namely, that a...
	"	line 8	....of a physical theory T, an...
	page 3	line 10	...epistemological structure...
	"	last line	..$\mathscr{S}_3$, mapping M into M*.
	page 4	line 3	...the hierarchy, i.e. modification...
		line 7	...(B. Russell)...
	page 5	line 2	...calculus of propositions, which...
	"	last line	...the investigations with which one is concerned....
par2{	"	paragraph 2	It has been...theorists, to the present, that.... was adequate. Acknowledgement of its use, or the use of any other formalism,....
	"	line 6	As such, the...
	"	line 9	....specifically, this...
	"	line 12	More recently, the validity of the Boolean formalism has been...
	page 7	line 6	...defined on any...
par.2→		line 1	...paper will take, as...
	page 8	last line	...after the fact a cochannel....
	page 9	line 7	...hence, that ....
	"	line 9	...and, in fact, will...
	"	line 12	...of meaning, and, if...
	page 11	line 4	...definition...
	page 12	line 4	...is similarly defined: the...
	"	line 6	...semantic subspace, we...
	"	line 21	...Then, when we...

par.2 {	page 17	line 3	...would permit speech...
		line 5	...until a more complete...system is supplied..
	page 18	line 1	...not split non-trivially into...
	page 20	line 9	...system (e.g. by.....)
	page 22	line 6	...each ensemble should...
par.2 {	"	line 9	...fact, coupled...
	"	line 10	...described, leads to...
par.3 →	"	line 4	...must proceed as a...
	page 24	line 12	...speech process, the...
	"	line 11	Thus, one will...
par. 2 {	"	line 12	... of units, regardless..
	"	line 13	...meaning, will be...
par.2 →	page 26	line 12	...of comprehension and...
	page 27	line 8	...physically correlated, to the...
	page 31	line 22	...conversation,but...
	"	line 27	...absolute propositional...
	page 32	line 4	...following experiment is suggested. A group of subjects are presented, either visually or orally, with a linguistic corpus chose at random from two corpi--one composed of propositionally complete statements and the other of propositionally incomplete statements. The subject may be asked to shadow the material or to answer questions demonstrating comprehension. Under circumstances in which the "listener" or the "reader" were given no other information, one would then expect better recogintion and less misinterpretation (fewer errors) with the subjects given a propositionally complete corpus than would result from a corpus composed of a large number of propositionally incomplete statements. The propositionally incomplete corpus could be...
	"	line 13	...likelihood,...
	page 33	line 9	...or emperically.
	page 36	line 2	...highly desirable at...
	page 37	line 1	...his debt and...
	"	line 6	...help, prior...

FORWARD

The discussions contained in this paper are unavoidably dependent upon previous arguments and examples. That the use of mathematical concepts may be unfamiliar to the reader is also unfortunate and to that end we suggest several sources for a reasonably concise exposition of the relevant concepts. Those references in the bibliography which are starred serve this purpose. It is also desireable that the reader who is interested in the mathematical formalism of the model presented here be familiar with the developments of RQ logic given in STC III, IV, and V by David Finkelstein. The paper which follows is largely concerned with the use of an established formalism in a new field and hence, it is at the metalanguage stage that most of the paper is presented. Further, it is hoped that the reader will have familiarized himself with the model of semiotic extension and ontogenesis given by McNeill as we believe the models to be in a set to subset relation. Examples and arguments given by McNeill are therefore highly relevant. The reader is thus forewarned: the proper understanding of this paper will not, in all likelihood, be an easy task, as it is of an extremely abstract nature and attempts a marriage of previously unaquanted disciplines.

I. Introduction

The delineation of the psychological aspects of the recognition and generation of linguistic phenomena as a predictive model is no mean task. In general one must have a clear understanding of what is meant by a predictive model and, at the same time, be able to state in explicit terms as many of the relevant and known phenomena as possible. Further, it is desirable to have the degree of insight which would permit one the ability to identify those elements of the given phenomenological data which are the best candidates to be considered as _a priori_ within the formalism. We emphasize here that the choice of what is to be considered as _a piori_ is purely arbitrary and is determined solely by an attempt on the part of the theorist to minimize the necessary assumptions and to maximize the explanatory power and ease of the model. In this section we shall endeavor to establish precisely what is meant by a predictive model and, in this manner, identify the direction to be taken in the remainder of the current paper with the hope that a predictive model of psycholinguistics will be the final result.

Inasmuch as the subject matter to be discussed will entail subject matter expected to be familiar to the reader, and because the establishment of an abstract formalism is in and of itself a most lenghty task, we will not overindulge in the practice of reviewing the considerable literature which details the known phenomena and theses of psycholinguistic research. Throughout this paper the reader is well-advised to keep in mind the formalism established for the so-called "static" properties of natural languages in a previous paper[1] "Quantum Logic and the Semantics of Natural Languages" as well as the definitions of terms given there. We will not misuse valuable space by reiterating those results here. Unless otherwise stated, terms will retain the definitions previously established.

Given some formalism of a theory it is necessary to provide an interpretation of that formalism and, in fact, just what it means to interpret a formalism.[2] Our discussion of this matter will be based upon the point of view established as the concept of logical empiricism which seems to be the most widely accepted; namely that a physical theory is a partially interpreted formal system.[3] We identify as constituents of a physical theory T an abstract formalism F and a set of rules of correspondence R. The abstract formalism F is a logical calculus devoid of any empirical meaning and containing descriptive terms as well. Inasmuch as F consists of both primitive and undefined terms (that is, terms derived from the former by logical rules), we may think of it as an axiomatized calculus. The set of rules of correspondence R serve to establish correlations between the nonlogical terms in F and observable phenomena or (preferably) empirical operations. We shall denote an interpreted formalism - that is, F+R - by F_R. The interpretation set (or rules of correspondence) R may also be called a metalanguage insofar as they are usually described in terms presumed to be familiar.

In addition to F_R, we require that the physical theory T have predictive power. That is to say, we require that T explicitly state the spatio-temperal interdependences that arise from a linear analysis of the observable quantities of the theory.This requirement can be met through the addition of some principle which serves to establish an internal coherence among the descriptive features of the theory. We call such a principle a model M of the theory defined as a fully-interpreted system whose logical structure is isomorphic to that of F_R. The epistomological structure of the model M, however, is such that logically prior propositions serve to determine the meaning of terms occurring at lower levels as compared to logically posterior propositions determining the meanings of terms occurring at higher levels in F_R. It is through the explanatory power of a model M that new avenues of research and solutions to intrinsic problems are suggested.

The time-dependent evolution of a model M is governed by the manner in which each of the constituent parts - both prior and posterior to the model - may be modified. The empirical nature of the model building process demands that, given any set of observations O which we wish to describe through some physical theory T, a modification $\varphi_1 : O$ into O^* through the addition and/or deletion of elements o must imply some modification $\varphi_2 : T$ into T^*. Similarly, any modification mapping T into T* implies a modification mapping F into F* and thus a modification φ mapping M into M*. Thus a modification in the observation set O or in any constituent of the hierarchy necessitates a modification in each posterior constituent of the hierarchy i.e. modification obeys the law of transitivity. In the reverse operation in which one has a given model M with modification φ_4:M into M* it must be noted that the inverse modifications φ_i^{-1} are not guaranteed. We may however approximate the modifications φ_i with modifications Φ_i having inverses Φ_i^{-1} with images which are locally isomorphic to those of the φ_i.

It is important to understand that the predictive power of a physical theory is largely determined by the existence of nonlogical terms or formulas in F which are not directly interpreted through R. These are called "theoretical" terms as divorced from empirical terms. Such terms are only implicitly or contextually defined through the role they play within the logical structure of F. It is always possible (B. Russel) to reduce theoretical terms to their constitutive elements, i.e. to observed objects or events or properties; to a set of propositions which contain only observational terms.

Having defined a predictive model in this manner we now seek to establish, from the known phenomenological set O a formalism F which will suffice to generate a proper predictive model M. The scientific method implies the initial assumption of a formalism or model, with modification appropriate to the empirical data which modifies the set O. As has been previously stated, a formalism is a logical calculus.

The most carefully delineated logical calculus at present is the calculus of propositions which may be described more clearly in terms of lattice theory (Birkhoff) as a complemented distributive lattice with a corresponding, abstract Boolean algebra defined on the quantities of the system. Thus, we first assume the calculus of propositions as a reasonable formalism in the pursuit of any physical theory for the sake of facillitating the investigations involved.

It has been the implicit assumption among psycholinguistic theorists to the present that the Boolean formalism was adequate to the extent that awknowledging its use or that of any other formalism is seldom explicit. This author has seen no example of such an awknowledgement in any text. As such the value of any such formalism and the models generated by it has not been recognized. However, there have been numerous circumstances under which the validity of this formalism has been brought under question; specifically this has been the case in investigations of the syntax of language (Carnap) and the logical structure of language (Chomsky, McCawley). More recently the validity has been placed under severe fire by various investigations in the field of artificial intelligence (Rulifson; Hendrix, Walker, Robinson) and in mathematics (Thom; Zeeman; Mesarović).

The logical hierarchy of a logical calculus is such that, in lattice theoretic terms, the primitive concept is that of an <u>element</u> which, through a concatenation process, generates a <u>chain</u>. A chain is a <u>partially ordered set</u> (p.o.s.)

and, a lattice is a partially ordered set with every pair of elements having a supremun (sup) and an infimum (inf). A lattice L need not be complete (i.e. every non-empty subset having a supremum and an infimum), finite (i.e. have a finite number of elements), complemented, distributive, nor modular (satisfies the modular identity). Thus it can be seen that the concept of a lattice is epistemically prior to that of a complemented, distributive (or Boolean) lattice. Furthermore, the modular identity is weaker than the distributive identity in that a distributive lattice is necessarily modular whereas the reverse is not given.

In the first paper of this series concerned with the implications of a non-Boolean formalism in the natural sciences,[4] it was demonstrated that there occur examples of non-distributive and, in fact, non-commutative elements in natural languages. That such examples occur under circumstances in which there exists a dependence on the implicit ordering relation is to be expected: the ordering relation is necessarily prior to the concepts of distributivity and commutativity. (It is interesting to note that, in general, the ordering relation is of a spatio-temperal nature, although this is not always the case.) Hence we are forced to the conclusion that the distributivity of the lattice which is to serve as isomorphic to the formalism of a physical theory concerned with the totality of that phenomenological set we call natural languages must be non-distributive if there exists one such example of non-distributive logic in any natural language. There exist other reasons to suppose the non-Boolean nature of linguistic logic. Inasmuch as natural languages are generative (i.e. self-organizing), it can be shown that it must be discrete - self-organization involves changes in the systems structure which can not be defined in any continuous space (Mesarović, 1962). Further, meaning itself can not be continuous. Given any two meanings, the supposition of continuity demands that a third meaning can be found between them. The exercise which demonstrates the falsity of this premise is so trivial as to be left to the reader in order not to be insulting. Lastly, some speech stimuli are known to be discrete (Liberman, Harris, Hoffman, and Griffith, 1957; Studdert-Kennedy, Liberman, Harris, and Cooper, 1970). The discretemess of language and meaning necessarily implies complementarity between some of the elements of the lattice, the failure of commutativity and distributivity[5], and hence, the failure of any Boolean structure as a model.

The current paper will therefore take, as its starting point, the assumption that the proper formalism for a psycholinguistic model wil be that of a complete, orthocomplemented, non-distributive, modular lattice (also known as quantum logic) with proper modification to take into account the dynamic aspects of the phenomenological set. Finkelstein[6] has been the prime developer of this formalism (as part of a model of physics) and calls it relativistic quantum logic or simply, RQ logic.

II. RQ Logic: A First Psycholinguistic Model

The transition from describing linguistic phenomena which are purely static (or which may be obviously approximated as such) to those which are of a time-dependent or dynamic nature (i.e. psycholinguistic phenomena) is a major theoretical step. The acquisition, generation, and recognition of linguistic utterances will be described in terms of a RQ-logical formalism. Psycholinguistic data necessitate this modification of the formalism established in a previous paper.[7] In turn, this modification will define certain modifications in a general linguistic model, containing the psycholinguistic model as a subset. Where appropriate to the current topic, we will point out these conclusions.

The concepts defined in Table I of Quantum Logic and the Semantics of Natural Languages and reproduced in the Appendix for the convenience of the reader, are given according to the classes of the abstract system which represents linguistic data. Inasmuch as that system was taken to be a static one, the classes of the system were taken to form a plenum. In the present paper we define a class by it's membership - that is, how we control membership in the class. Such a definition is an operational one which takes into account the obvious dynamic character of psycholinguistic data. The classes of the system are thus shown to form a plexus - the model will be a model of processes. We call a logical device which regulates the classes of a system in advance a channel and one which regulates membership after the fact a

That class membership of semantic systems is a dynamic process is readily seen by the changes which typically occur in dictionary entries over a period of years. That we were previously at fault in not considering this dynamic character of linguistic utterances is not in dispute - we took linguistic utterances as occurring in zero time as a first approximation; hence that the logic used in describing them need not be concerned with time-dependent ensembles or processes. We can no longer maintain this view and in fact will take as <u>a priori</u> the fact that semantic digits form a quantum network of semantic processes. If language consists of quantum units of meaning and if their generation or recognition is time-dependent, it follows that each unit must occur in some finite temporal period. In the current paper, by a semantic digit we will mean a quantum semantic process.

In the development of the dynamics of quantum logic which was pursued by Finkelstein in his paper Space-Time Code III, several new concepts are introduced which we find useful here. The first is that of a <u>stator</u>, a logical-mathematical object which serves to decribe a system, being a "vector" in a Hilbert space associated with a quantum system. Each stator serves to determine a projection and thus a class of the system. A change in the class of the system corresponds to a change in the projection operator and hence a different stator. A stator is a property. It is an adjective, not a noun, in a very abstract sense. It can be thought of as a static; logically operational object inasmuch as systems which contain a certain membership class are characterized by the properties consistent with a given stator.

The second concept of particular interest is that of a <u>plexor</u> which serves to describe the general role of a stator in a quantum plexus in much the same manner as a tensor is a generalized vector. Just as a tensor maps a vector space with associated scalars into a second geometric object, a plexor maps one stator into a second stator. Assigning a plexor to a system of processes is tantamount to assigning an ordering relation. Prior to the assignment no geometrical relations exist. The system is said to be a <u>pregeometry</u>. Finally, we say that a quantity <u>exists</u> or <u>is possessed by</u> a system if the classes, as the values <u>v</u> of the quantity (with values in a set V) range over V, are exhaustive and mutually exclusive. This is equivalent to defining the quantity operationally in showing how to control the values of the quantity.

A quantum logic which deals with quantum processes does not use all the standard relations as defined in Table I. It is a non-trivial property of the world that all the concepts of language can be expressed in the language of the RQ-logical relations of inclusion $\subset$ and exclusion $\perp$. In terms of a process network or plexus, an object is either a member of a class or it is not. Psycholinguistic systems, as compared to linguistic systems which are concerned almost exclusively with simple systems as defined in Table I, are concerned with both compound and complex systems and such is the topic of the current paper. We are now interested in the properties of natural languages which the theory predicts for compound and complex systems; that is, how the theory predicts such systems will be formed out of simple systems and therefore what macroscopic effects might result from such rules of generation.

We end with the following definitions for a complex variable: it is the trace over a two-sequence space with an operator which is a map of the space into the series with respect to some basis (in the sense of coordinate basis). More complex units are built up from digits (primitive processes). We will explore this concept further in the next section. Finally, we assume for the model a relativisticly causal ordering relation.

III. The Interpretation

Several results of the preceding formalism are immediately apparent concerning the generation of linguistic utterances. In that section we defined a complex variable. Accordingly, a complex semantic variable is simarly defined: the trace over a two-sequence semantic space with an operator which is a map of the semantic into the series with respect to some coordinate basis.Larger semantic units are built up from semantic digits through a process of concatenation. Given some semantic suspace, we start with a simple system and create a compound system following the procedures outlined in Table I. From this compound system we may then define, as an interpretation, some semantic object which is the trace previously described. The operator defined over the basis is thus used to generate a meaning interpretation. In order to generate a complex system from this binary one, the same operator is used again in a reiteration process. However, since the basis over which the operator is defined changes as the system becomes more complex, we end by having a concatenation process which proceeds unit by unit. This process is more readily delineated by the projection operator of a stator, in this case a semantic stator. We express information about the system by means of the projection operator. Then when we change information about the system, we also change the projection itself. This is equivalent to replacing one stator by a second.

We shall call the projection operator of a stator defined on some semantic space a <u>syntactor</u>.

We may treat the basis space of the system in two ways - either staticly or dynamicly. If the basis space is static, then we have effectively given it a boundary and reduced its extension to finite range. Thus any change must occur as the result of some operator outside that boundary. Any descrip-of the semantic space is therefore static as well and must postulate systems external to itself to allow for the dynamic aspects of language. The dynamic case includes the contextual environment as part of the total semantic space and treats as a subspace that part which undergoes a time-dependent evolution. Under such conditions the generation of an utterance begins with some given semantic subspace. The stator determines a given projection without unique closure - that is to say, the boundary of the subspace is not clearly defined. Such a condition is equivalent to the syntactor being a degenerate eigenfunction. Then any such semantic subspace which is linguisticly determined may be said to be ambiguous. This is the general form of the static definition of ambiguity which was given in a previous paper. The result of such an ambiguity is the ability - indeed the necessity - for the system to take some semantic digit which is contiguous as operationally defining the boundary and thus providing closure.

However, this process is reiterative. The new subspace defines new information on the system and thus a new syntactor.

With the new syntactor, a new projection is defined and thus, the boundary is no longer <u>necessarily</u> closed.

At the macroscopic level this is equivalent to the generative process being continually altered by percepts and by environmental constraints generally. We do not make a distinction here between internal and external constraints, nor the time-dependence or time-independence of such constraints. In a pregeometric view such as this, meaning, whether from memory, the local environs, or from ones internal states and emotions, are simply portions of the semantic subspace. Indeed, the generative process may be described as a concatenation process which is context sensitive. Most often this will show up as a situation in which speech is generated one word at a time and in which the context (including all stimuli, both internal and external) will determine the outcome of the next step of generation.

Because the level in the hierarchy of semantic structures at which the so-called generative mechanism is proposed to operate is so deep, we are at a stage of formalism which is prior to that which allows for any distinction between the direction of semantic flow, across any arbitrary boundary designated to distinguish the interior of the human system from the exterior, is recognizable or even possible. The concept of locality, built into the logic of the system through the relativistic causal ordering relation, precludes any ability <u>on the part of the generative mechanism</u> to distinguish between semantic subspaces which derive their class membership from sensory processes or from internal states, processes, and feedback - either through channels or cochannels. That is to say, the mechanism may operate in an identifying mode with equal efficiency. The difference is a relative one: in the generative mode there exist contextual constraints which prevent closure for the speaker whereas in the identifying mode they may be taken as continually providing closure for the listener. These two conditions are operationally equivalent.

The microscopic description of the concatenation process implies three different methods of statistical analysis of the semantic complexes: those which are formed by the sequence, the series, and the set of semantic digits as defined in Table II of the Appendix. These ensembles may be analyzed respectively be Maxwell-Boltzman, Bose-Einstein, and Fermi-Dirac statistics. We suggest that, under appropriate and specific circumstances, each of these statistics will be found to be evidential at the macroscopic level. This topic will be explored in more detail in the following section since the results are largely of a predictive nature.

The acquisition of linguistic utterances is now well-defined within the current formalism. It is the time-dependent semantic subspace with observables which are the subset of the complex variables of the system with cardinality which forms a sequence in time which is monotonic and increasing. Given such a definition of language acquisition we feel well justified in proposing the following tenet: that all human beings are born with the faculty for that which has been called language acquisition (unless physically impaired) and that that those linguistic observables known as syntax and morphology are macroscopic evidences of the intrinsic ordering relation involved both in semantic processes <u>and in all physical processes</u>. We make explicit now several assumptions that are inherent to the observation of language acquisition: first, that the observed system is <u>defined</u> i.e. that a boundary is recognized which separates the system from its environment; second, that there exists a time-dependent subspace (semantic) of increasing extent i.e. that there is a transfer of linguistic information across the said boundary; and third, that the system is alive i.e. that it is self-organizing and hence is discrete and non-singular.

IV. Results

There arises the question of .what one might expect to observe regarding language acquisition, speech production, speech perception, speech recognition, speech comprehension, linguistic performance and competence, and linguistic determinism and relativity. In this section we will undertake to answer those questions inasmuch as the current model makes adequate predictions concerning these topics. We can expect from the onset that several experimental studies might thus be suggested. However, it should be understood that the results of such studies will, in general, only supply information which will permit that a refinement of the model obtain. The further questions which can be expected are at present unanswerable and we take this as being further evidence that the modeling procedure has been a fruitful one.

In the last section we have claimed that a child, by nature of its existence and definition, must have inherent abilities which would premit speech acquisition and production. While a great many questions concerning behaviour must await answering until we have a complete model of the human system supplied by further research, the fundamental processes involved in language acquisition may be outlined at this time. The term "language acquisition" is found, by this author at least, to be antiquated and rather misleading. When observing the system of child plus environment from a process frame of reference - that is, as a plexus - it is apparent that a portion of the system undergoes a time-evolution which is linear and causal. If the system did not spit non-trivially into disjoint subspaces consisting of time-dependent and time-independent semantic ensembles, there would be no means of identifying the system in a time-independent fashion. The time-evolution of the system, part of which is the behaviour called language acquisition, is described by those subspaces of ensembles which are time-dependent. In the current interpretation, time-dependence simply means a concatenation process of binary digits - that is, a sequence of quantum processes. In refering to the linguistic portion of the system, these are then semantic processes. We can describe the concatenation conveniently if we know the ordering relation and we have then the abstract algebraic entity which we have called a syntactor.

For the sake of explication, suppose that the firing of a neuron may be taken to represent a semantic digit.[8] Neuronal structures permit branching of considerable complexity, and so we may not predict from a knowledge of what neuron has fired which neuron will fire next or at least be activated. It would be a further necessity to know simultaneously the states of all the neurons which synapse with it. What determines the firing order - the concatenation order? Two factors are of importance: first, the states of impingent neurons which in linguistic terminology and with the current model represent the contextual environment; second, the ordering relation. We claim that the neuronal structures which are present in the human brain are 'ordered' in such a manner as to demand an isomorphism between neural and linguistic structures. We have considerable justification for this claim. If firing is indeed representative of a semantic digit (and we will claim in a latter paper on the neuropsychology of the model that linguistic structures must operate at a level as deep as neural structures based on the current evidence), then the structures which correspond to linguistic phenomena and those which correspond to neural phenomena must be isomorphic. Since it is the ordering relation which determines the topology (structure) of an ensemble, we may conclude an equivalence of ordering relations.

The reader is reminded at this point that <u>all</u> physical processes are subject to the ordering relation, whether they be semantic, electrical, or material. Hence, the time-evolution (growth) of a neuron may also be described in such terms. We are now faced with just such an unanswerable question as was earlier anticipated. When the psycholinguist observes the time-evolution of a linguistic ensemble (for example, language acquisition), is he observing the physical growth of neurons, the changing of neural inhibition potentials, or perhaps the restructuring of neurons? It has been demonstrated that these neural processes are to some degree coupled - each may trigger the other. Hence it seems unlikely that a better answer is available than a general one, namely that language acquisition is limited by the potential growth and modification of neurons and neural structures, manifested physically by a change in <u>effective</u> neural structure, and observed only through the limitations - both time-dependent and time-independent - which are imposed by the experimental arrangement which includes the experimenter and which is a semantic subspace with operators of its own. That the general answer should be one concerned primarily with limitations on the modeling process should not be surprising. We are attempting an expression of a linear, causal ordering of the system when we question language acquisition and its multiple relationships to neural structures, potentials, and growth. If the system is multiply-dependent, then it can not admit of such an ordering in and of itself. Only by restricting the variability of some portion of the system (i.e. by treating neural structure, potential, and growth as a single variable associated with a complex ensemble and constraining its degrees of freedom to some extent), may we <u>artificially</u> produce such a result. I call an operator on a semantic system an <u>inhibitor</u> if such a result obtains. Since inhibitors are "supplied" by that semantic subspace defined as the experimentor or observer in any situation, the investigation of possible inhibitors which would yield, in union, a complete description of the system under study, is an introspective one and thus one can only determine the tautologies of the system. These then, in an operational sense, are the limitations of the system deductively.

What then is the proper description of language acquisition within the current model? We shall claim that the acquisition of a semantic digit is equivalent in a physical sense to the ordering of a neural element. This is, in turn, according to the above comment, equivalent to the time-dependent structure, growth, and inhibition potentials of neurons. We have previously argued that neural evolution is a strictly determined physical process and therefore language acquisition is governed by physical laws. Language acquisition, taking these presumptions into account, is that segment of the time-evolution of a semantic subspace which under the influence of a local (contiguous) semantic space yields the observable phenomena of verbalization. We further define verbalization as a transfer of semantic digits of a linguistic nature across the boundary separating the subspace (organism) and the subspace orthocomplement (environs).

This model of language acquisition suggests several observable speech learning processes. First, one will expect that the local environs may be used to optimize the acquisition of semantic digits and that this optimization must be dynamic in nature. Second, since language acquisition is dependent on both neural structures and the local contextual environment, it follows that the potential to achieve verbal expression should in fact precede the actualization to some extent. Hence, by providing both verbal and sensorally rich environments for the infant which are strongly coupled, one might expect to minimize both the time required to induce the neural changes appropriate to acquisition and the lag time between potential and actual verbalization. Third, one might expect that words which decompose into few semantic digits would be more readily acquired than those which require higher ordered complexes. That this is a reasonable expectation is readily seen considering the degree of neural organization which would be required as a correlate to such acquisition. Fourth, more commonly experienced linguistic units (words, semantic digits, etc.) will be acquired more readily in as much as inhibition potentials and growth of neurons is modified by continued excitation of a given neural pathway. Fifth, inasmuch as the model does not differentiate between semantic digits originating from and used for tactile, oral, visual, or other sensory process and, in fact, claims that such digits will be contiguous in a semantic space representative of the organism (this being the case since all senses are simultaneously operative).[9]

We can expect that each acquired linguistic unit will have several discernable components. The complex semantic ensemble should consist of both time-dependent and time-independent features, these being the equivalent of McNeill's analysis of utterances into action/event/object/property/entity/state/location features. Further, each encemble should be strongly correlated to the physical behaviour of the system - these are an inseparable part of the meaning of the ensemble. This fact coupled with the time-evolution of a semantic subspace as earlier described leads to the prediction of semiotic extension á la McNeill.

In describing language acquisition we have inadvertantly given the foundations for an explanation of speech production, perception, recognition, and comprehension. We have stated in the last section that speech production must procede as a concatenation process. We now repeat that explanation from the viewpoint provided in the present context (psycholinguistic observation) with special emphasis on the observable predictions of the model.

The concatenation of semantic digits must proceed in a manner consistent with the ordering relation and thus with the logic of the system under consideration. Furthermore, one must recognize the contextual environment as influencing the possible choice of semantic digits in the concatenation process. As previously stated, if the complete semantic space is considered, then the choice of semantic digits is determined within the limits of a degenerate eigenfunction. Each digit in the chain is followed by a digit which is locally contiguous and which belongs either to the semantic subspace internal or external to the organism. The choice will be determined as an instantaneous maximization of the propositional completeness of the utterance. Where there exists a true degeneracy in the maximization a choice must take place. This choice is a dynamic operator and qualifies in every aspect as that abstract notion traditionally called <u>will</u>.

What does this description predict as observables in speech production? First, that speech production is <u>always</u> context sensitive. By context we imply a semantic subspace which is determined by both the environment and the internal states of the organism. A conclusion of this line of reasoning is that altering one or more of a psycholinguistic subjects environment, speech history, internal states, or personal history will appear to causally alter the subjects speech production. In other words, what one says depends on the context, how one feels, how one thinks, and one's memory of related subjects. Second, the model predicts that speech production is an observably concurrent process; that it occurs so rapidly as to imply incompatibility with models of speech production that involve transformational processes. Third, the model predicts that certain speech errors should occur with statistical distributions that are non-classical. In Table II we defined the three different objects which we may mean by an ensemble of n semantic digits: sequence, series, and set. We have taken as prior the notion of sequence - an ordered n-tuple of semantic digits isomorphic to the macroscopically observable semantic object, in this case the speech sequence. If we take speech errors to be random distributions of semantic digits which do not fit the implied order of the speech process. The collection of such errors will then occur with statistical distribution appropriate to the set of such semantic digits as the "errors" represent, namely Fermi-Dirac statistics.

Fourth, there is the rather immediate prediction that, due to the fact that meaning is discrete and the "choice" of sequential semantic digits is random, the occurrence of words will follow a Poisson Interval Distribution. This fact will be the more obvious with frequently used semantic units - words such as "if", "and", "the", etc. Fifth, since the occurrence of such units retains its order despite being taken out of sequence, we thus have a series of semantic objects and hence it follows that for an ensemble of n such objects, the frequency of occurrence will follow Bose-Einstein statistics. Sixth, it should be remembered that the occurrence of semantic units will be classical in general. Thus one will expect that the statistics describing the occurrence of units regardless of meaning will be a Maxwell-Boltzman distribution.

Seventh, there is the prediction that speech production is of a fundamentally ambiguous nature. Given any "completed" statement, it can be shown to admit of at least two interpretations. The degree to which this is observably true will be dependent upon the degree to which the entire semantic universe is taken into account. If this were done under ideal conditions the result would be an expression of a semantic uncertainty relationship. Until such a time as the cardinality of the semantic digits forming an ensemble can be empiricly identified, the proper formulation of such a relationship will not be possible. However, cybernetics has managed to provide us with some useful clues.[10] Information has been related to energy via relation 1 below in which v_{max} is the bandwith of the channel in bits/sec, h is Planck's constant, c is the speed of light, and S/N is the signal to noise power ratio.

$$E_{max} = mch^{-1}\log_2(1+S/N) \qquad (1)$$

Now for relatively low temperatures the energy cost per bit is greater than or equal to kT where k is Boltzman's constant and T is the temperature of a system of mass m. Substituting these relations into the Heisenberg uncertainty yields 2.

$$v_{max} = 2m \times 10^{47} \text{bits/sec} \qquad (2)$$

Taking into account the temperature of the human system and thus of thermal noise and quantum mechanical fluctuations, the final limitation on speech production is given by 3.

$$v_{max} = 4.3 \times 10^{12} \text{ semantic digits/sec} \qquad (3)$$

It is then theoretically possible to use these last two relations in order to prepare experimental procedures which will serve as partial verification of the concepts presented in this paper. Namely, one would expect to be able to measure the bit content of some prepaired text and then, by presenting it very rapidly to a subject and slowing it incrementally, determine the comprehension threshhold.

The linguistic processes involved in the transfer of information from the environment to the subject has traditionally been divided up into three areas of inquiry, each describing a process temporarily ordered with respect to the other two. Researchers have assumed that the subject first percieves the input, recognizes it, and that comprehension then follows. That perception precedes recognition and comprehension is readily demonstrated once we have been predisposed to a temporal analysis of linguistic data flow. There are many examples of subjects indicating that a datum has been percieved but not recognized - even when the datum had been percieved previously. Distinguishing the processes of coprehension and recognition is not so trivial a task since we are dependent upon the subject in any given experiment to report the difference subjectively; we want to know if the datum has been simply remembered as having been previously experienced or has it been organized with respect to previous data for the first time.

We argue here that there is no meaningful operational distinction between the processes of recognition and comprehension, and that, in fact, the distinction from perception is even more ambiguous. Perception is the process whereby a datum actually enters the semantic space of the subject; it is usually taken as a process description of the senses. The senses, in the current model, are then operationally either channels or cochannels inasmuch as they provide partial regulation of class membership. Thus we can describe perception in the following manner: perception is the process observed when a logical object which we have called a semantic ensemble (a percept) appears in the time domain to be taken as part of a semantic subspace (the subject) through a given channel or cochannel (one of the senses), defining a new syntactor on the subspace and thus new conditions for closure ("thought"). New conditions for closure may elicit semantic ensembles which are physically correllated to the physical behaviour of the system (e.g. speech, movement, etc.) or semantic ensembles which are uncorrelated to the physical behaviour in any obvious manner (we observe this effect introspectively as thought - and in some cases observe it externally as well).

Now it begins to appear that we have the essentials of the processes called recognition and comprehension within the act of perception alone; we need only account for the observable distinctions, whether subjective or objective, between them. First of all, recognition implies that there exist identical semantic ensembles both internal and external to the subject. Further, and this is not a trivial implication, it implies that either (1), the semantic ensembles external become internal to the subject and some sort of comparison takes place, the information being no longer available exposternally (we call this the postulate of semantic conservation), (2) that the postulate of semantic conservation is satisfied and that no transfer actually occurs, or (3) that information is transfered and is always available externally. This last possibility is the most frequently assumed - that information concerning an object is not-conserved and is always duplicatable. That this is not the case is readily demonstrable from the physics of known physical mechanisms associated with observable systems. Observation alters a system to some finite degree and thus the information about the system is also altered. Thus we feel greatly justified in claiming faith in the postulate of semantic conservation.

But what of the process of comparison that has been implied in standard concepts of the process of recognition? Is it a necessary complication of the process description? We think not. The observable distinctions between perception and recognition are understood if one hypothesizes the existence of a syntactor which expresses a relative null change in the semantic space of the subject. Such a syntactor, which I call here the <u>null syntactor</u>, may be illustrated by a physical example. Suppose one has a computer (analogous to the subject) to which one gives a command (percept). If the command is contained in the memory (semantic space) of the computer, a machine language instruction (syntactor) will cause an execution to occur. If the command is not contained in memory, the computer will respond via a special machine language instruction (null systactor) with something like "Illcom" or "illegal command". This response is equivalent to the more human "I don't understand" or "Would you repeat that please" or "I don't recognize that."

Hence we can describe the recognition process in terms of that for perception described above if we generalize our means of describing the observed differences between these processes. We may similarly account for the process of comprehension by taking into account the operational meaning of comprehension. Defining comprehension thusly implies semantic connectivity between ensembles which are contained by the semantic ensembles to be understood. In other words, we are defining the lattice structure which is delineated by the ordering relation of the semantic ensembles thus entailed. Such ordering is particularly apparent when the effects of comprehension are observable - namely when one can test for it. Tests for comprehension, by definition, must involve some physical action. That such action can be speech shows that the connectivity between semantic ensembles has been established. This is operationally the same process as we have described for acquisition.

Thus we see nothing new or special between the processes of perception, recognition, or comprehension. In each, there is an influx of information in the form of a semantic ensemble and the generation thus of a new syntactor for the semantic space. The only differentiation between the processes described is whether that syntactor is the null syntactor (or something similar) or not.

At this point one should again take note that the process involved in speech recognition is the same as that in speech production. The only difference is the apparent source of the semantic ensemble responsible for changing the syntactors of the system. And we reiterate: at the process level such a distinction is not possible. A psycholinguistic model of this sort claims that, so long as one could not observe the source of information to and from the mind of the subject, one could not determine whether the subject is listening or speaking. Two other points are of special note as well. First, one would expect that the null syntactor might produce a physiological correlate that could be observed via the EEG for instance. This state should be a transient one and might be similar to long-term sensory deprivation states. Second, one might be able to further justify the postulate of semantic conservation. Since semantic systems are context sensitive[11], this postulate becomes tantamount to claiming a form of semantic uniqueness for all semantic ensembles. Once again we have derived a statement concerning the fundamental ambiguity in identifying semantic systems. In principle one might be able to make a measurement of this uncertainty in meaning and work backwards toward a proper statement of conservation via cybernetics in the manner previously indicated.

The question of comparing linguistic performances with one's linguistic competence naturally arises in any investigation of psycholinguistics and this is no exception. Modeling linguistic competence here is equivalent to explaining the process of language acquisition (which we have) and language retention. In as much as language retantion is an important half of the question we will briefly mention a few key points without proof or argument as such arguments are beyond the scope of this paper. However, a full model of memory will be undertaken in a later paper concerned with neurophysiological models predicted within the framework of this and the previous paper. Essentially, we will claim that no spatially local record of past events and information need be postulated and that the only distinciton between "past" and "present" semantic digits and ensembles is supplied by the ordering relation. In fact, without the ordering relation defined in a particular way, memory as divorced from percept is an arbitrary distinction. We claim that only the ordering relation need be concurrent in the semantic space of the subject and that, by definition, this is built into every semantic digit.

Linguistic performance can be modeled essentially by describing how errors can occur within the speech production process and this topic is divided into three questions: how do errors in speech occur, how do errors in remembering occur, and how do errors in aquisition occur. If, as was previously suggested, linguistic processes occur at the neural level of organization, then we are looking for ways in which firing, innibiton potential, or growth of neurons is altered from the norm. That these problems apply to all three questions of linguistic performance is obvious. However, "errors" can also occur when there are ambiguities generated. The concatenation process occurs in a manner which is locally context sensitive. In many cases, a degenerate eigenfunction will allow the production of a new semantic ensemble which is only locally meaningful in the context of, say, a conversation but which is truly an error at a higher level of the semantic hierarchy.

When we speak of ambiguities, we are refering to a concept of degree of ambiguity in as much as we claim that if communication is to continue, absolute closure and therefore absolut propositional completeness can not occur. In fact, recent experiments suggest that it is the ability to err, which allows for the great computational abilities of the human system.[12] In order to demonstrate the connection between errors and propositional completeness, the following experiment is suggested. Under circumstances in which the "listener" or "reader" were given no other information, one would then expect better recognition and less misinterpretation than would result from a corpus composed of a large number of propositionally incomplete statements is the presented material were composed of complete statements. The propositionally incomplete corpus could be prepared in such a manner as to be complete sentences and to contain the same information as the contrasting corpus of complete propositions. Further, one would expect that the speech errors which would occur would have a higher liklihood of occurence precisely in those locations where there existed the greatest degeneracy of the eigenfunction.

V. Conclusions

Having completed the model and it's implications, it remains to point out the circumstances under which evidence can be demonstrated for the correctness of the model - that is, the utility of the model. That the model is consistent will be a test of how well we have exercised the ability to produce reasonable conclusions from the first principles. In only rare cases have we borrowed postulates or axioms which were not a part of the initial formalism and these we have either attempted to justify logically or empiricly. In only one case have we depended upon arguments introduced from conjecture (this being the assumption that semantic processing occurred at a level equivalent to that of the neuron in the hierarchical structures of the brain) and this was done for the purposes of clarification, the actual process formalism and the associated conclusions being independent of whether a neuron or an electron or a neural net are responsible for representing a semantic digit.

We shall not indulge in the rather lengthy pursuit of examples in psycholinguistic research to justify the predictions of the model and the utility of it. Rather, we now seek to satisfy the reader by demonstrating that the model is able to subsume an isomorphism of another model for which empirical findings have been carefully listed. Our model has been concerned largely with processes at the microscopic level of psycholinguistics. Nevertheless, certain of the structures (concepts) have and must have applicability at the macroscopic level if the everyday features of psycholinguistic phenomena are to emerge in the classical limit of the theory.

The model which has been chosen to be incorporated at the macroscopic level is that of McNeill (1975). In the first chapter of that text, the author listed seven main statements or arguments of the remainder of the book which were subsequently followed in later chapters by justification both by empirical and logical argument. These were as follows:

1) The earliest organized utterances of children are based on action schemes. Thus interiorization is claimed.

2) The claim of undifferentiated sensory-motor concepts.

3) Interactions of speech and thought withsensory-motor concepts lead to indexical signs.

4) Word order acts as a kind of image or picture of the speaker's thoughts as they are organized for speaking.

5) The detailed claim of semiotic extension.

6) The importance of context - this including the speaker's meaning and perception of context.

7) Even speech disturbances follow the syntagmatic organization of the utterance.

It should be readily obvious that the present model is fact claiming much the same arguments although from a different level in the hierarchy and for reasons of the directly deducible abstract logical structure of natural languages. Hence it seems that, where we are involved with the same issues, the only difference has been that of terminology and method of derivation of the results. Where McNeill and others have spoken of the problem of serial order, we have refered to a causal linear ordering relation. The concept of a syntagma corresponds directly to a particular kind of semantic ensemble. Inasmuch as the model we have produced has not relied heavily on the empirical evidence of psycholinguistics, it has been a general one. Hence, definitions of concepts such as context have been extended as has that of ambiguity. Further,it must be remembered that the current model is a process model and thus conceptual differentiation such as the categories of location, action, object, etc. tend to be coalesced into a single logical object with differentiations being concerned with time-dependence or independence.

We know of no contradictions between the two models and therefore, it seems that the empirical evidence for one is the evidence for the other as well. And there is evidence for the model we have presented that is of importance. On page 24 we explored some of the statistical relationships predicted by the theory. Such relationships do infact occur. G. Herdan (1960) finds that ferquently used words follow a Poison Interval Distribution and that number/frequency distributions follow Bose-Einstein statistics. W.Feller (1968) finds that misprints follow Fermi-Dirac statistics. Also it is of note that G.K.Zipf (1949) has demonstrated that number/frequency distributions are universal in word use statistics.

Some of the other predictions of the theory are also known to be true. Some await further study. All in all it seems that the pursuit has been fruitful and that the formalism which we have attempted to establish might prove useful in presenting the problems of psycholinguistic research in clearer terms.

Admittedly the terms used and the methods may not be familiar to those in the field. Nevertheless, it is highly desireable at this stage in the development of psycholinguistics to attempt to interface with the mathematical methods and formalisms that have been established in other empirical pursuits. We believe that, though the learning may be painful, it is the only way we will arrive at a truly predictive model of such phenomena. We hope that the reference to and dependence on other sources for the understanding of this paper will not prove too much of a burden for the reader. It is the intent of this author to attempt a more concise and self-contained exposition at a later date.

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Footnotes

1. Briefly, the static properties of language are to be equated with the properties of language which are investigated in linguistics. Given a corpus of linguistic data we seek to discover the synactic and semantic properties of language and to model these in a single theory. The previous paper - ref.52 - gave evidence for the contention that these properties could be summarized in a model of quantum semantic objects with an internal and intrinsic ordering relation, this ordering relation giving rise to the macroscopic observable we call syntax.
2. See ref. 39, page 10.
3. Ibid.
4. We refer here to the intention to continue this work in a series of papers which we hope will eventially be a book. Tentatively we refer to this subject as Humanistic Physics. The steps to be completed are an investigation of generalized communications theory, psychology, and philosophy, with alternative interpretations of some principles of physics within the formalism once the isomorphism between physical processes and a subset of psychological processes has been given.
5. This being an extension of the quantum postulate of physics, generalized to encompass similar systems in its scope.
6. See ref. 24 - 30.
7. See ref. 52
8. This supposition is justified by several facts: ref.82, and the fact that the required physical factors are observed in neural structures.

9. This presents an n-dimensional map problem. Given **m**-different types of semantic digits (operationally different in 3-space in that they are derived from different sensory inputs) what is the minimum n-space required to provide a map for all digits so that each is contiguous with m-1 different digits? This then is the minimum n-space for an isomorphism of the human brain.

10. See Bremerman in ref. 88.

11. A result of ref. 52 also - the definition given there is implied.

12. See ref. 40.

TABLE I

Simple Systems

For quantum systems, the algebra of a system S^A is irreducible being the algebra of all maps of an underlying innerproduct space $\underline{I}(S)$. In this part all concepts are relative to one implicit system S.

Class (of a system S): = projection (quantity equal to its * (adjoint) and square) in S^A; subspace P,Q,... of the underlying linear space of S^A.

$P \subset Q$, P is *included* in Q (of classes P, Q): = the basic eigenvalue equation $PQ=P$; the subspace inclusion $P \subset Q$.

$\underline{I}$ and $\emptyset$, *universal and null class*: = quantities 1 and 0; $\underline{I}(S)$ and the 0 vector, as subspaces of $\underline{I}(S)$.

$P \cup Q$, P or Q (*adjunction*): = $\sup(P,Q)$; span $P \cup Q$ (the set join of two subspaces never being required).

$P \cap Q$, P and Q (*conjunction*): = $\inf(P,Q)$; subspace meet $P \cap Q$.

Q is a *complement* of P: $=P \cup Q = \underline{I}, P \quad Q=\emptyset$; Q is a complementary subspace to P.

$-P$, the *negation* of P: $=1-P$; orthogonal complement of subspace P.

$P \perp Q$, P excludes Q: $=PQ=0$; P and Q are orthogonal subspaces.

P comQ, P is compatible or *commutes* with Q: a basis exists for $\underline{I}(S)$ adapted to both subspaces P and Q.

f(S), a *coordinate* f of S: =map $f: S \to \underline{C}$; spectral family $dP_f(z)$ of subspaces, z a complex variable. Any coordinate f may be represented by a coordinate quantity $f=\int z \, dP_f(z)$, where the projection-valued measure $dP_f(z)$ is defined by the algebra map $f_A: \underline{C}^A \to S^A$.

$P \subset_1 Q$, P is just included in Q: $P \subset X \subset Q$ if and only if $P=X$ or $X=Q$; $Q=P\cup$ one additional 1-space.

$|P|$, the measure of P:= the length m of a chain $0 \subset_1 P_1 \subset_1 \ldots \subset_1 P_m = P$.

$\underline{o}$, a singlet;= projection $\underline{o}$ with measure = 1; a ray or 1-space of $\underline{I}(S)$.

If G is any group of maps $g: S \to S$ and G_A is the group of induced algebra maps, we can then define as follows:

S/G,,S over G: =the algebra $S^A \backslash G_A$, the collection of those quantities of S^A invariant under G_A; the algebra of operators on $\underline{I}(S)$ commuting with all members of the (unitary) group G. Even if S is a quantum system, S/G generally is not.

S G, S under G: = the algebra S^A/G_A resulting from S^A by identification with respect to G_A; the subspace of $\underline{I}(S)$consisting of all fixed points under G.

Let P be a class of S:

$S \backslash P$. S under P, the restriction of S to P: = the algebra PS^AP taken with the +, X, * of S^A but with the new unit P; the subsystem defined by a subspace $P \subset \underline{I}(S)$

The system 1: = the system whose algebra is $\underline{C}$; system with a one-dimensional Hilbert space. The system 1 is both a classical syatem (commutative) and a quantum system (irreducible).

In quantum logic the distributive law is weakened to the form

If $a \subset c$, then $a \cup (b \cap c) = (a \cup b) \cap c$. Note that it is self-dual: replacing $\subset, \cap, \cup$ by $\supset, \cup, \cap$ merely replaces a,b,c by c,b,a. It also follows that $(a \cup b) \cap c = (a \cup b) \cap (a \cup c)$.

For quantum assemblies, it is not generally true that $a \supset b = -a \cup b$.

Compound Systems

S+T, the *sum* of S and T: = the direct-product algebra $S^A T^A$, in which the two algebras S^A and T^A commute; the direct product Hilbert space $\underline{I}(S)X\underline{I}(T)$. Similarly for IIS_i. Associative and distributive laws hold.

S $\underline{R}$ T, a *binary relation* $\underline{R}$ between systems S,T: =a class of ST; subspace of the direct product $\underline{I}(S)X\underline{I}(T)$.

S-T, *similar* systems S,T: = two systems S,T provided with an equivalence map $e:S \to T$ (map with inverse); two Hilbert spaces with a unitary $e:\underline{I}(S) \to \underline{I}(T)$. We designate corresponding projections in S,T by P(S)-P(T). *Replicas* of a system S are similar systems obtained from S by attaching labels,e.g. S_1-S_2

S=T: = for similar systems S-T, the class $U_{\underline{a}}\underline{a}(S)\underline{a}(T)$, the union extending over all singlets $\underline{a}(S)$-$\underline{a}(T)$; symmetric subspace of the direct product.

Reflexive relation: =relation S $\underline{R}$ T with $(S=T) \subset (S\ \underline{R}\ T)$; subspace of $\underline{I}(S)X\underline{I}(T)$ including the symmetric subspace.

R^T, the transpose of R: $=eXe^{-1}(R)$ where $e:S \to T$ is the equivalence map of S-T and $\underline{R}$=S $\underline{R}$ T.

Symmetric relation: =relation $S=S^T$.

Transitive relation: =relation $\underline{T}$ with $S_1\ \underline{T}\ S_2, S_2\ \underline{T}\ S_3 \subset S_1\ \underline{T}\ S_3$

Functional relation: =relation S $\underline{F}$ T= $U_{\underline{a}}\underline{a}f_A(\underline{a})$,where $\underline{a}$ ranges over the singlets of S, and $f:S \to T$ is a map; the graph $U_{\underline{a}}(\underline{a}Xf_A(\underline{a}))$ of an algebra map $f_A:T^A \to S^A$.

seq_2S, the *2-sequence* of S's: =the product S_1S_2 of two replicas S_1-S_2 of S; the ordered pair of two S's.

dia_2S, the diagonal 2-sequence of S's: = $seq_2S \backslash (S_1=S_2)$, the restriction of S_1S_2 to the class $(S_1=S_2)$; the subspace of symmetric tensors in $\underline{I}(S_1)X\underline{I}(S_2)$

Let G be the symmetric group on two similar systems, S_1-S_2. ser_2S, the 2-series of S's: $=\mathrm{seq}_2S/G$ with G as above; the subalgebra of $S^A X S^A$ invariant under transposing; the direct sum of the subalgebras of the symmetric and antisymmetric subspace of $I(S) X I(S)$.

Complex Systems

seq_nS, the n-sequence of S's: $= \Pi_M S_M$ ($M=1,\ldots,n$), where S_M-S are similar systems; the direct product of n replicas of $I(S)$.

seqS, the sequence of S's:= ${}_n\mathrm{seq}_nS$ ($n=0,1,\ldots$); the Maxwell-Boltzman Fock space over $I(S)$, with the number operator N as superselection rule.

dia_nS, the diagonal n-sequence of S's: $=\mathrm{seq}_nS$ ($S_1=\ldots\ldots=S_n$); the space of symmetric tensors of degree n over $I(S)$.

diaS, the diagonal sequence of S's: = ${}_n\mathrm{dia}_nS$ ($n=0,1,\ldots\ldots$); the Bose-Einstein Fock space over $I(S)$, with the number of systems N as superselection.

Let G be the symmetric group on the systems in a sequence seqS. Then we define as follows:

serS, the series of S's: =seqS/G; the subalbegra of seqS^A invariant under G.

Table II

Quantum Statistics

sequence the tensors on I_a	The n sequence of c's, an ordered n-tuple of objects isomorphic to c, is the object c^n whose universal class is I^n, the n-th power of I, with cardinality $/I^n/ = /I/^n$. The generic sequence of c's is the object seq c, which is an n-sequence for some n, the disjoint union seq c = $U_n c^n$. The cardinality of seq I is infinite if /I/ is greater than 0.
series the symmetric tensors on I_a	The n series of c's, an unordered n-tuple of objects isomorphic to c, is the object c^n with universal class I^n obtained from I^n by identifying with respect to permutations of the n objects, or is the symmetrized n-th power, with cardinality $/I^n/ = (i+n-1)!/(i+1)!n!$ with $i = /I/$. The generic series of c's is the object ser c, which is an n series for some n: ser c $= U_n c^n$. The cardinality of ser c is infinite if /I/ is greater than 1.
set the antisymmetric tensors on I_a	The n set of c's, a set of n c's, is the object c^n with universal class I^n obtained from I^n by identifying with respect to permutations of the n objects and deleting sequences with two or more identical elements, or is the antisymmetrized n-th power, with cardinality $/I^n/ = i!/n!(i-n+1)!$. The generic set of c's is the object set c which is an n set for some n: set c $= U_n c^n$. The cardinality of set c is then 2^i, and set c is usually written 2^c.

The substitution of classical objects c with quantum objects q makes I become a Hilbert space and makes the above descriptions of seq, ser, and set become valid descriptions of the ensembles of "Maxwell-Boltzman" objects, "Bose-Einstein" objects, and "Fermi-Dirac" objects, respectively. In this process all products of sets are replaced by direct products of Hilbert spaces, unions are replaced by direct sums. Discovering the statistics of an object is equivalent to discovering whether it is a seq, ser, or set ensemble.

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